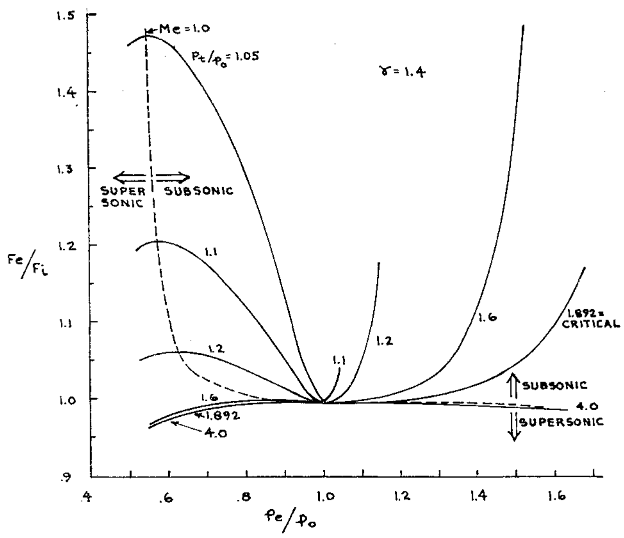

Fig. 1 Effect of nozzle exit pressure on thrust.

define the thrust ratio of actual to ideal thrust as

$$F_e/F_i = [m_eV_e + (p_e - p_0)A_e]/m_eV_i \tag{1}$$

For a perfect gas with constant specific heat ratio we find

$$F_e/F_i = (V_e/V_i)\{1 + [(p_e - p_0)/(p_e\gamma M_e^2)]\} \tag{2}$$

If we further assume isentropic flow without heat addition and use standard one-dimensional relationships,[2] for example,

$$P_t/p = \{1 + [(\gamma - 1)/2]M^2\}^{\gamma/(\gamma-1)} \tag{3}$$

We can express Eq. (2) in terms of the actual and ideal exit Mach number,

$$\frac{F_e}{F_i} = \frac{M_e}{M_i}\left(\frac{1 + \frac{\gamma-1}{2}M_i^2}{1 + \frac{\gamma-1}{2}M_e^2}\right)^{1/2} \times \left(1 + \frac{1 - \left\{\frac{1 + [(\gamma-1)/2]M_i^2}{1 + [(\gamma-1)/2]M_e^2}\right\}^{\gamma/(1-\gamma)}}{\gamma M_e^2}\right) \tag{4}$$

where M_i is the exit Mach number for expansion to ambient pressure and M_e is for expansion to the actual exhaust pressure. It should be noted that the exit area for subsonic flow must be varied to maintain the same mass flow.

From Eq. (4), F_e/F_i may be plotted as a function of M_e and M_i, but a more convenient presentation is in terms of the nozzle pressure ratio (P_t/p_0) and the exit pressure ratio (P_e/p_0). This is shown in Fig. 1 with a dashed line to illustrate the regions of subsonic and supersonic flow at the nozzle exit. These results reveal some interesting trends. First we note that above or to the right of the dashed line the exit flow is subsonic. In this region, the thrust ratio increases continuously with exit pressure for nozzle pressure ratios (P_t/p_0) less than critical (1.892 in this case). This is typical of the ground-effect machine in which a very large thrust results with a low exit velocity. At higher nozzle pressure ratios, the thrust increase is less pronounced.

Secondly, we note that for exit pressures lower than atmospheric, the thrust of a subsonic flow again rises, reaching a peak value (at $M_e = 1$) for nozzle pressure ratios less than critical. For supersonic exit flow (to the left of dashed line), the thrust drops from the peak because the increase in exit velocity is more than offset by the lowered pressure.

The effect of exit pressure is most marked when the basic nozzle pressure ratio is small. Thus, for example, with a nozzle pressure ratio of $P_t/p_0 = 1.2$, a local exit pressure reduction of 40% or a pressure increase of 10% will each produce a thrust increase of about 6% above the ideal theoretical value.

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Rapid Estimation of Wing Aerodynamic Characteristics for Minimum Induced Drag

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Nomenclature

AR	= aspect ratio = (span)²/wing area
C_{LD}	= design lift coefficient
$C_{L\alpha}$	= lift coefficient per degree, $(dC_L/d\alpha_L)$
C_{mD}	= pitching moment about wing apex at design lift condition
C_{m_0}	= pitching moment at zero lift for $(\epsilon_T/\alpha)_{OPT}$ condition
$C_{m_{01}}$	= pitching moment coefficient about wing apex for $C_L = 0$ and $\epsilon_T = -1°$
$C_{m\alpha}$	= pitching moment coefficient about wing apex per degree, $(dC_m/d\alpha_L)$
M	= Mach number
α	= total angle of attack = $(\alpha_0 + \alpha_L)$, deg
α_0	= zero-lift angle of attack for $(\epsilon_T/\alpha)_{OPT}$ condition, deg
α_{01}	= zero-lift angle of attack for 1° tip washout, deg
α_L	= flat-plate angle of attack needed to develop desired C_{LD}, deg
β	= Prandtl compressibility factor = $(1 - M^2)^{1/2}$
ϵ_T	= twist at tip (washout is negative), deg
$(\epsilon_T/\alpha)_{OPT}$	= tip twist angle/angle-of-attack ratio for minimum induced drag (elliptical spanwise loading)
η	= nondimensional semispan location (0 at root, 1 at tip)
Λ	= sweep angle at quarter chord, deg
Λ_β	= compressible sweep angle = arc tan[(tanΛ)/β], deg
λ	= taper ratio = tip chord/root chord

Introduction

THE data presented in this note were generated as part of a large parametric study to determine basic aerodynamic characteristics over a wide subsonic Mach number range for uncambered, flat-plate wings having trapezoidal planforms (parallel root and tip chords) with straight leading and trailing edges. This type of wing was considered because of its desirability from a manufacturing standpoint. The amount of linear-lofted twist required for minimum induced drag for such wings was also determined. Body interference and viscous effects were not considered.

Discussion

The object of this note is to provide information that can be used to determine rapidly the complete wing characteristics for a near-optimum planform-twist combination at any specified subsonic flight condition. The lifting-surface computer program used in this study is described in Ref. 1. Briefly, the wing loading is numerically approxi-

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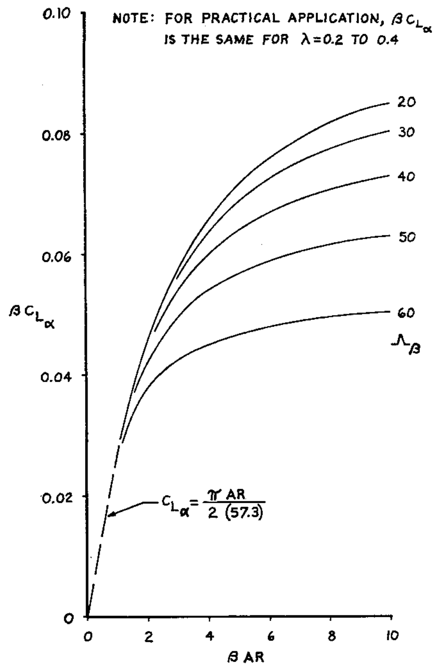

Fig. 1 Variation of lift curve slope with βAR.

mated by a vortex lattice that is sized by imposing the boundary condition of no flow through the wing surface at a selected number of control points equal to the number of vortices used. This method is based on the work of J. L. Crigler of NASA, a portion of which is discussed briefly in Ref. 2.

To compress the large amount of data generated, all wings are related through the Prandtl transformation in terms of β, AR, and Λ_β.[3] Through this transformation, a single set of curves can be used to represent the aerodynamic parameters for any wing in subsonic flow. Individual wing characteristics may be determined by first finding β's from

$$\beta = \tan\Lambda/\tan\Lambda_\beta \tag{1}$$

at which the plotted values of Λ_β correspond to the desired wing sweep angle, Λ. Using these values of β, the variation of $C_{L\alpha}$, α_{0_1}, $C_{m\alpha}$ and $C_{m_{01}}$ with βAR can be determined from

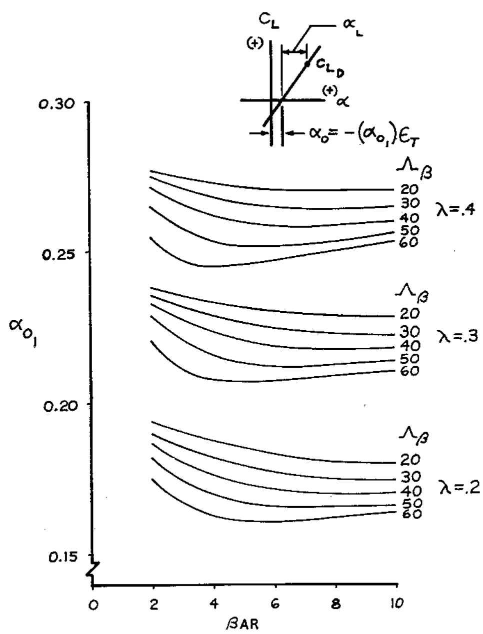

Fig. 2 Variation of zero-lift angle of attack with βAR for 1° tip washout.

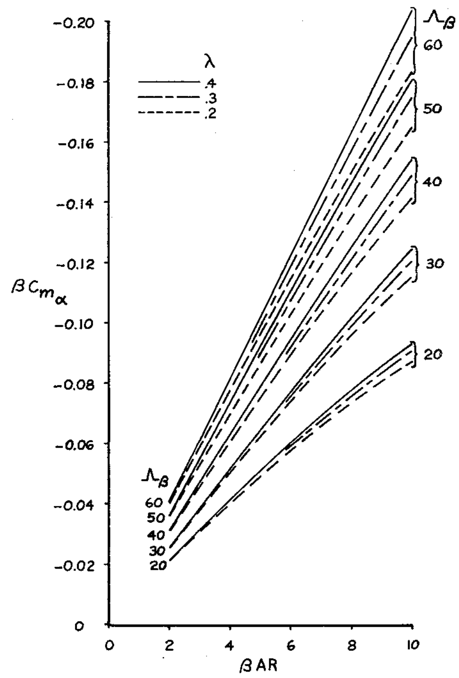

Fig. 3 Variation of pitching moment coefficient slope with βAR.

Figs. 1–4, respectively. By cross-plotting this information vs Mach number, complete wing aerodynamic characteristics at any desired subsonic Mach number can be obtained.

The amount of linear-lofted twist needed for minimum induced drag (elliptical spanwise loading) at a given angle of attack has also been determined. For trapezoidal planforms with straight leading and trailing edges, the linear-lofted twist angle at any spanwise station is given in Ref. 4 by

$$\epsilon = \epsilon_T\eta\lambda/[1 - \eta(1 - \lambda)] \tag{2}$$

Spanwise twist variations obtained from this expression were used in the lifting-surface computer program for several values of tip washout at a fixed angle of attack to determine the minimum induced drag. The tip twist angle/angle-of-

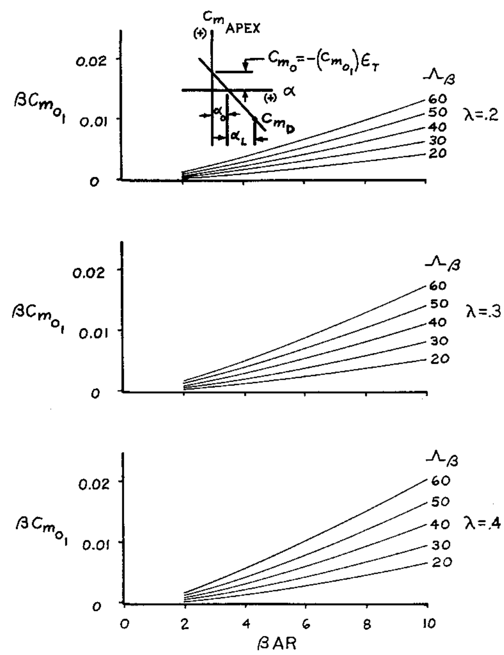

Fig. 4 Variation of zero-lifting pitching moment with βAR for 1° tip washout.

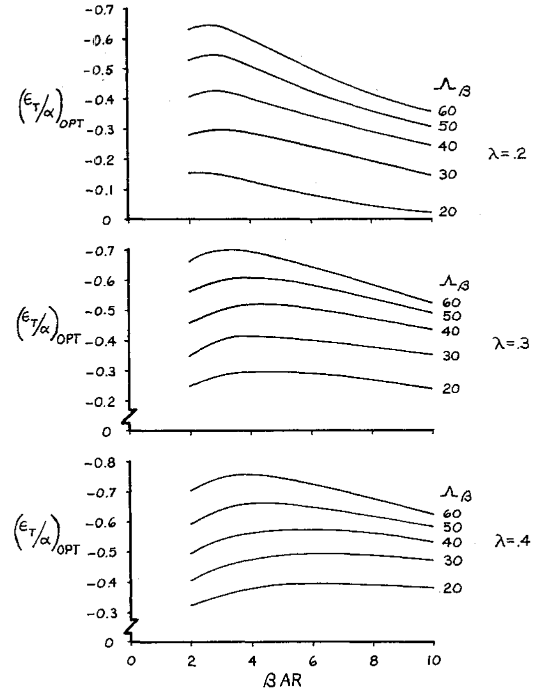

Fig. 5 Variation of optimum tip twist angle/angle-of-attack ratio with βAR.

attack ratio, which produces minimum induced drag for a particular wing, is defined as $(\epsilon_T/\alpha)_{\rm OPT}$. Although this optimum cannot be defined with great precision (because of curve fits within the computer program used to calculate wing efficiency), the variation near the optimum is small, so that slight uncertainties will probably not be important for most practical applications. Figure 5 shows the variation of $(\epsilon_T/\alpha)_{\rm OPT}$ with βAR for twisted, trapezoidal planform wings with taper ratios of 0.2, 0.3, and 0.4.

To design a wing of given planform to have a specified C_{LD} and minimum induced drag, values of α_L, ϵ_T, and α_0 must be known. These can be determined as follows:

$$\alpha_L = C_{LD}/C_{L\alpha} \tag{3}$$

where $C_{L\alpha}$ is obtained from Fig. 1;

$$\epsilon_T = \alpha_L(\epsilon_T/\alpha)_{\rm OPT}/[1 + \alpha_{0_1}(\epsilon_T/\alpha)_{\rm OPT}] \tag{4}$$

where α_{0_1} and $(\epsilon_T/\alpha)_{\rm OPT}$ are obtained from Figs. 2 and 5, respectively; and

$$\alpha_0 = -(\alpha_{0_1})\epsilon_T \tag{5}$$

Thus all quantities needed for wing design are determined.

This note provides information that can be used to estimate rapidly the complete wing aerodynamic characteristics for any uncambered, flat-plate trapezoidal planform wing at subsonic Mach numbers including the linear-lofted twist needed to produce minimum induced drag for a specified lift coefficient. Data obtained from this procedure provide a good starting point for detailed consideration of more complex wing configurations.

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Rolling Moment Due to Sideslip of Delta Wings

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Nomenclature

C_L	= lift coefficient
C_l	= rolling moment coefficient
β	= angle of sideslip, rad
Λ	= angle of sweepback, deg
Γ	= dihedral angle, deg
$C_{l\beta}$	= $\partial C_l/\partial\beta\|_{\beta=0}$
$(\partial C_{l\beta}/\partial C_L)_0$	= $\partial C_{l\beta}/\partial C_L\|_{C_L=0}$
$(\partial C_{l\beta}/\partial C_L)_A$	= contribution of aspect ratio to $(\partial C_{l\beta}/\partial C_L)_0$
$(\partial C_{l\beta}/\partial C_L)_\Lambda$	= contribution of sweepback to $(\partial C_{l\beta}/\partial C_L)_0$
A	= aspect ratio

An extensive literature survey has been conducted to correlate existing experimental data with various predictions of the rolling moment due to sideslip for delta wings.

For incompressible flow the rolling moment due to sideslip for an isolated wing is given by the equation

$$C_{l\beta} = C_L\left[\left(\frac{\partial C_{l\beta}}{\partial C_L}\right)_A + \left(\frac{\partial C_{l\beta}}{\partial C_L}\right)_\Lambda\right] + \Gamma\left(\frac{\partial C_{l\beta}}{\partial \Gamma}\right)$$

The portion of $C_{l\beta}$ which is a linear function of lift coefficient is composed of two terms. The first term, $(\partial C_{l\beta}/\partial C_L)_A$, which is a function of aspect ratio and taper ratio, arises from two sources: the mutual induction of the leading and trailing wing panels, and the windward wing tip, which becomes effectively a leading edge with high negative pressures at the wing tip. The second term, $(\partial C_{l\beta}/\partial C_L)_\Lambda$, is the contribution of sweepback to $(\partial C_{l\beta}/\partial C_L)_0$. This contribution to rolling moment arises entirely from the difference in effective sweep angle and aspect ratio of the leading and trailing wing panels in sideslip.

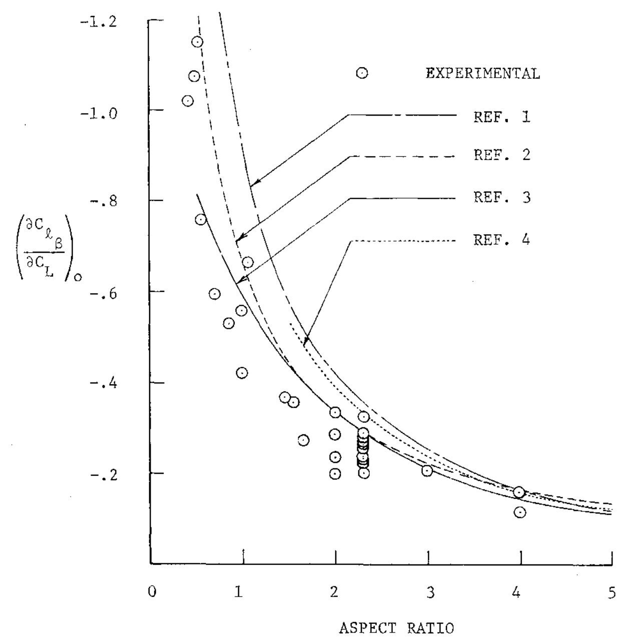

Fig. 1 Experimental and predicted values of $(\partial C_{l\beta}/\partial C_L)_0$ for delta wings.

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